Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 3B Further Statistics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $*$ The answer is printed on the paper
   - ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Qu	Scheme	Mark	AO
1 (a)	$E(X) = [-1\times 0.2 + 0 + 1\times 0.2] + 3\times 0.25 + 5\times 0.25$ [= 2]	M1	1.1b
	$E(X^2) = (-1)^2\times 0.2 + 0 + 1\times 0.2 + 3^2\times 0.25 + 5^2\times 0.25$ [= 8.9]	M1	1.1b
	[Var(X) = 8.9 – 4 =] **4.9**	A1	1.1b
		(3)	
(b)	$E(X^4) = (-1)^4\times 0.2 + 0 + 1\times 0.2 + 3^4\times 0.25 + 5^4\times 0.25$ [= 176.9]	M1	2.1
	Var(X^2) = " 176.9" – " 8.9" ^{2}	M1	1.1b
	= 97.69 awrt **97.7**	A1	1.1b
		(3)	
		(6 marks)	

	Notes
(a)	1st M1 for an attempt at E(X), at least the final 2 products seen or an answer of 2 2nd M1 for attempting a correct expression for E(X^2), at least 3 non-zero products seen A1 for 4.9 or exact equivalent
(b)	1st M1 for attempting a correct expression, at least 3 non-zero products seen (implied by 176.9) 2nd M1 for a correct method, ft their 176.9 and their 8.9 but must be intending E(X^4) and E(X^2) A1 for 97.69 or awrt 97.7

Qu	Scheme	Mark	AO
2 (a)	**(i)** 0.20901… awrt **0.209**	B1	3.4
	(ii) 0.30844.. awrt **0.308**	B1	1.1b
		(2)	
(b)	H_0: $\lambda = 2.4$ (or $\mu = 6$) H_1: $\lambda \neq 2.4$ (or $\mu \neq 6$)	B1	2.5
	[E = no. of errors] E ~Po(6)	M1	3.3
	P($E \leqslant 1$) = 0.0174 <u>or</u> P($E \leqslant 2$) = 0.0620 **and** P($E \leqslant 11$) = 0.980 <u>or</u> P($E \geqslant 12$) = 0.0201	M1	3.4
	Critical region: $E \leqslant 1$ or $E \geqslant 12$	A1	1.1b
		(4)	
(c)	[P(Type I error) = 0.0174 + 0.0201 =] **<u>0.0375</u>** (Calc gives: 0.017351… + 0.0200919… = 0.037443…)	B1ft	1.2
		(1)	
		(7 marks)	

	Notes
(a)	1st B1 for awrt 0.209 2nd B1 for awrt 0.308
(b)	B1 for both hypotheses correct in terms of λ or μ (allow $\lambda = 6$ etc) 1st M1 for selecting the correct model. Sight or use of Po(6) 2nd M1 for use of the correct model with two probs correct to 2.s.f. (accept P($E \geqslant 12$) = 0.02) Must see attempt at lower **and** upper limit. Probabilities may be seen in (c). A1 for correct critical region (both parts). Allow $E \leqslant 1$ and $E \geqslant 12$ or $E \leqslant 1$, $E \geqslant 12$ etc Writing CR as probability statements is A0 **NB**: Completely correct CR implies M1M1A1 <u>SC: 1-tailed test</u> B0 as hypotheses are incorrect M1 for sight or use of Po(6) M1 (dep on H_1) for sight of P($E \leqslant 1$) = 0.0174 or P($E \geqslant 11$) = 0.0426, in line with their H_1 A1 for CR: $E \leqslant 1$ or CR: $E \geqslant 11$, in line with their hypotheses
(c)	B1ft for 0.0375 or 0.0374 <u>or</u> summing their two appropriate probs (ft their CR) **NB:** If candidate uses a 1-tailed test, this mark cannot be gained

Qu	Scheme	Mark	AO
3 (a)	H_0: There is no association between the colour chosen and year group (allow use of "independence" instead of association)	B1	2.5
	H_1: There is some association between colour and year group	**(1)**	
(b)(i)	Need lowest row total and column total so "Black" and "10-12"	B1	2.2a
(ii)	With expected frequency $\frac{68\times 19}{240} = 5.3833\ldots$	B1	1.1b
		(2)	
(c)	$v = (5-1)\times(3-1) = \mathbf{8}$	B1	3.4
	cv of $\chi_8^2(1\%) = \mathbf{20.090}$	B1ft	1.1b
	(significant) evidence of an association between colour chosen and year group	B1ft	2.2b
		(3)	
		(6 marks)	

Notes

(a) B1 for both hypotheses in context. Must mention colour and year group at least once
NB: condone use of related/correlated/linked etc if recovered with independent/associated

(b)(i) B1 for a choosing "Black" and "10-12" **with** some correct reasoning mentioning row and column totals (allow clear equivalent, e.g. 'lowest total frequencies')

(ii) B1 for a correct expression or awrt 5.38

(c) 1st B1 for a correct calculation or answer of 8. May be implied by sight of cv of 20.09
2nd B1ft for 20.090 (accept 20.09) or a correct ft 1% cv using their df and correct to 2 d.p.
3rd B1ft for a correct conclusion in context (ft their cv)
E.g. "students in different years (tend to) have different favourite colours"
Do not allow contradictory statements. Condone 'related/relationship' in part (c)
NB: We ignore their hypotheses when marking (c)

Qu	Scheme	Mark	AO
4 (a)	$X \sim \text{Geo}(\frac{1}{6})$ accept in words: <u>geometric</u> distribution with $p = \frac{1}{6}$	B1	3.3
		(1)	
(b)	$P(X \leqslant 3) = 1 - P(X > 3)$ <u>or</u> $P(X = 1) + P(X = 2) + P(X = 3)$	M1	1.1b
	$= 1 - \left(\frac{5}{6}\right)^3$ or $\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{91}{216}$ (*)	A1cso	1.1b
		(2)	
(c)	$E(X) = 6 \quad \text{Var}(X) = \dfrac{\frac{5}{6}}{\left(\frac{1}{6}\right)^2}$ $[=30]$	M1 A1	3.4 1.1b
	$\bar{X} \approx \sim N\left(6, \left(\sqrt{\frac{30}{64}}\right)^2\right)$	M1 A1	3.3 1.1b
	$P(5.6 < \bar{X} < 7.2) = 0.68064\ldots$	A1	1.1b
		(5)	
(d)	$H_0: p = \frac{1}{6} \quad H_1: p < \frac{1}{6}$	B1	2.5
	Probability approach: $P(X \geqslant 16)$ $= P(X > 15) = \left(\frac{5}{6}\right)^{15} = 0.0649\ldots$ | CR approach: $\left(\frac{5}{6}\right)^{c-1} < 0.05$ or $\left(\frac{5}{6}\right)^{d} < 0.05$ with use of logs CR: $X \geqslant 18$ or $X > 17$	M1 A1	3.4 1.1b
	(Not significant) insufficient evidence that <u>probability</u> is $< \frac{1}{6}$ or (Not significant) insufficient evidence that dice is <u>biased</u>	A1	2.2b
		(4)	
		(12 marks)	

Notes

(a) B1 for both "geometric" or "Geo" **and** correct parameter of $\frac{1}{6}$. Probability must be seen in (a)

(b) M1 for a correct method, may be implied by a correct expression
A1cso* for a correct solution leading to printed answer with no incorrect working seen

(c) 1st M1 for correct use of formula for E(X) <u>or</u> Var(X)
1st A1 for both correct must see value of 6 for E(X) and at least correct formula used for Var(X) the value of 30 may be implied by 2nd A1
2nd M1 for use of CLT to get a normal distribution with correct mean and variance ≠ their 30
Condone use of X (or their letter) instead of $\bar{X}$ or seeing $N(6, \sqrt{\frac{30}{64}})$
2nd A1 for a correct normal distribution stated or used. May be implied by correct answer.
3rd A1 for awrt 0.681 but allow 0.68 or 0.680 if correct distribution is seen

(d) B1 for both hypotheses correct in terms of p
M1 for sight or use of $P(X \geqslant 16)$ (may be implied)
or correct expression with logs (use of logs may be implied by 17.43…)
1st A1 for 0.065 or better but accept 0.06 if a correct expression is seen, **or** correct CR
2nd A1 for a correct conclusion mentioning <u>probability</u> or <u>biased</u>
Condone 'evidence suggests <u>probability</u> is equal to $\frac{1}{6}$'

NB: Must have correct probability or correct CR to gain final A1.
NB: Send responses comparing 0.935 with 0.95 to review

Qu	Scheme	Mark	AO
5 (a)	$H_0: p = 0.03 \quad H_1: p > 0.03$	B1	2.5
		(1)	
(b)	$D_{100} \sim B(100, 0.03)$ $[P(D_{100} \geqslant 5) = 1 - P(D_{100} \leqslant 4)]$	M1	3.3
	$= 0.18214\ldots$ awrt **0.182**	A1	1.1b
		(2)	
(c)	$P(D_{80} \geqslant 5) + P(D_{80} = 4) \times P(D_{80} \geqslant 1)$	M1	2.1
	$= 0.09279\ldots + 0.12654\ldots \times 0.91255\ldots$	A1	1.1b
	$= 0.20826\ldots$ awrt **0.208**	A1	1.1b
		(3)	
(d)(i)	Test **A**: [$X \sim B(100, 0.06)$ $P(X \geqslant 5) =$] 0.72322… awrt **0.723**	B1	1.2
(ii)	Expected number = $80 + 80 \times P(Y = 4)$ with $Y \sim B(80, 0.06)$	M1	3.4
	$= 94.87\ldots =$ **95**	A1	1.1b
		(3)	
(e)	Tests of similar size and power but Test **B** involves sampling fewer components so would advise to use test **B**.	B1	2.4
		(1)	
		(10 marks)	

Notes

(a) B1 for both hypotheses in terms of p or π

(b) M1 for sight or use of the correct model. Allow for 1 – 0.81785… or 1 – 0.91916..
A1 for awrt 0.182

(c) M1 for a correct expression for required probability. May be implied by 1st A1
1st A1 for a correct numerical expression – values to 2 s.f. or better
2nd A1 for awrt 0.208

(d)(i) B1 for power of test **A** = awrt 0.723
(ii) M1 Correct expression with $Y \sim B(80, 0.06)$
or for use of B(80, 0.06) to obtain a probability of 0.814 or 0.186
Implied by sight of 95 or better
A1 for 95 (accept awrt 94.9)
NB: May see $0.814 \times 80 + 0.186 \times 160 =$ awrt 94.9 or 95, which is M1A1

(e) B1 for a choice backed up by a suitable reason:
If concluding **B**, they need to mention <u>similar power</u> and a <u>smaller sample size</u>
If concluding **A**, they need to mention <u>smaller size</u> and <u>greater power</u>
NB: We do not ft the candidate's incorrect values for size or power

SC Use of Poisson approximation
(b) Allow M1 for Po(3) and A1 for answer of 0.1847 or better
(c) Allow M1 and 1st A1 for awrt 0.21 but 2nd A0 (Po(2.4) gives 0.2099…)
(d)(i) Allow B1 for 0.7149 or better from Po(6)
(ii) Allow M1 for expression (answer should be 94.56…) A0 for answer.

Qu	Scheme	Mark	AO
6 (a)	$[G_X(t) = (4-3t)^{-\frac{1}{2}} \Rightarrow]$ $G'_X(t) = \frac{3}{2}(4-3t)^{-\frac{3}{2}}$; [So E($X$) =] $G'_X(t) = \frac{3}{2}$	M1; A1	2.1; 1.1b
	$G''_X(t) = \frac{27}{4}(4-3t)^{-\frac{5}{2}}$; so $G''_X(1) = \frac{27}{4}$	M1; A1ft	2.1 1.1b
	Var(X) = " $\frac{27}{4}$ " + " $\frac{3}{2}$ " − $\left(\text{" } \frac{3}{2} \text{ "}\right)^2$; = **6**	M1; A1	1.1b 1.1b
		(6)	
(b)	Using Maclaurin:		
	$\frac{G''_X(0)}{2!} = \left\{\frac{1}{2}\right\} \times \text{" } \frac{27}{4} \text{ "} \times \frac{1}{32}$		
	$[P(X=2) =]$ $\frac{1}{2} \times \frac{27}{4} \times \frac{1}{32} \left[= \frac{27}{256}\right]$		
	$[P(X=0) + P(X=1) =]$ " $\frac{3}{2}$ " $\times \frac{1}{8} + \frac{1}{2}$		
	P($X \leqslant 2$) = $\frac{203}{256}$		
	Using Binomial:		
	$[G_X(t) =]$ $\frac{1}{2}\left(1 - \frac{3}{4}t\right)^{-\frac{1}{2}}$	M1	2.1
	$\frac{1}{2}\left(\ldots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{3}{4}t\right)^2 + \ldots\right) = \left[\ldots + \frac{27}{256}t^2 + \ldots\right]$	A1	1.1b
	$\frac{1}{2}\left(1 + \frac{3}{8}t + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{3}{4}t\right)^2 + \ldots\right)$	M1	2.1
	P($X \leqslant 2$) = $\frac{203}{256}$	A1	1.1b
		(4)	
(c)	$\left[G_W(t) = G_{X_1}(t) \times G_{X_2}(t) \times t =\right] \frac{1}{\sqrt{4-3t}} \times \frac{1}{\sqrt{4-3t}} \times t$; $G_Y(t) == \frac{t}{4-3t}$	M1; A1	3.1a/2.1 1.1b
	$G_Y(t) == \frac{\frac{1}{4}}{1-\frac{3}{4}t}$ or $G_Y(t) == \frac{\frac{1}{4}t}{1-\frac{3}{4}t}$; ***Y* ~ Geo($\frac{1}{4}$)**	M1 A1	2.1 3.2a/2.2a
		(4)	
(d)	P(Y > 6) = $\left(1 - \frac{1}{4}\right)^6$; $= \frac{729}{4096}$ = 0.177978… awrt **0.178**	M1 A1	3.4,1.1b
		(2)	
	Notes	**(16 marks)**	
(a)	1st M1 for attempt to differentiate leading to $k(4-3t)^{-1.5}$; 1st A1 for E(X) = 1.5 or exact equivalent 2nd M1 for attempting to differentiate again leading to $m(4-3t)^{-2.5}$ 2nd A1ft for $\frac{27}{4}$ or correct ft from their k provided both Ms are scored 3rd M1 for a correct method for finding Var(X); can ft their $\frac{3}{2}$ and their $\frac{27}{4}$ 3rd A1 for 6		
(b)	1st M1 for Maclaurin to find P(X = 2) condone " $\frac{27}{4}$ " $\times \frac{1}{32}\left[= \frac{27}{128} \text{ or } 0.2109375\right]$ **or** putting in the form $a(1-0.75t)^{-0.5}$ 1st A1 for a correct unsimplified prob for P(X = 2) (may be in binomial expansion) Allow 0.105(468…) 2nd M1 for use of pgf to find P(X = 1) and P(X = 0) <u>or</u> attempt 1st 3 terms of bin expansion 2nd A1 for $\frac{203}{256}$ or exact equivalent.		
(c)	1st M1 for using product of pgf or multiplication by t 1st A1 for correct unsimplified form of pgf 2nd dM1 (dep. on 1st M1) for attempting to convert pgf to form given in the formula book **or** for stating Geometric alongside a correct PGF for Y (may be unsimplified) 2nd A1 for correctly deducing the distribution of Y as geometric with p = 0.25 [may be seen in (d)] **NB:** Final A1 dependent on previous marks being awarded.		
(d)	M1 for attempting to use geometric formula or their pgf (using correct coefficients) ; A1 for awrt 0.178		

Qu	Scheme	Marks	AO
7(a)	[X = no. of prizes Sam wins] $X \sim B(20, 0.2)$	M1	3.3
	$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 0.58855\ldots$ awrt **<u>0.589</u>**	A1	1.1b
		(2)	
(b)	[Y = no. of game when Tessa wins her 4th prize] $Y \sim \text{negB}(4, 0.2)$	M1,	3.3
	$P(Y = 20) = \binom{19}{3} 0.2^3 0.8^{16} \times 0.2$, $= 0.043639\ldots$ awrt **<u>0.0436</u>**	A1	1.1b
		(2)	
(c)	S = no of prizes Sam wins in n games $S \sim B(n, 0.2)$	M1	3.1b
	Profit $= (2 + 4 + 6 + \ldots + 2S) - n = \frac{S}{2}\left[4 + 2(S-1)\right] - n$	M1	2.1
	$= S^2 + S - n$	A1	1.1b
	$E(S) = 0.2n$ and $Var(S) = 0.2 \times 0.8n$	M1	3.1b
	$E(S^2) = 0.16n + 0.04n^2$ or $\frac{1}{25}(n^2 + 4n)$	A1	1.1b
(*)	So expected profit for Sam is $\frac{1}{25}(n^2 + 4n) + \frac{1}{5}n - n = \frac{1}{25}(n^2 - 16n)$	A1cso	3.2a
		(6)	
(d)	<u>Using profit expression:</u> / <u>Using a listing method</u>	M1	3.1b
	Require $P(S^2 + S - n \geqslant 0)$ / Indicates that 4 wins is first non-loss		
	Solving quadratic, leading to $S = \ldots$ / $P(S \geqslant 4)$	M1	2.1
	$P(S \geqslant 4)$ where $S \sim B(15, 0.2)$ / $P(S \geqslant 4)$ where $S \sim B(15, 0.2)$	M1	1.1b
	$= 0.35183\ldots$ awrt **<u>0.352</u>**	A1	1.1b
		(4)	
(e)	T = game on which Tessa wins her rth prize $T \sim \text{negB}(r, 0.2)$ or $r(r + 1)$	M1	3.3
	Profit $= (2 + 4 + 6 + \ldots + 2r) - T = r(r + 1) - T$	A1	1.1b
	Tessa's expected profit $= r(r + 1) - E(T)$	M1	3.4
	$= r^2 + r - \frac{r}{0.2} =$ **<u>$r^2 - 4r$</u>**	A1	1.1b
		(4)	
	Notes	**(18 marks)**	

(a) M1 for selecting a suitable model (B(20, 0.2)
A1 for awrt 0.589

(b) M1 for stating correct negative binomial or $19C3p^3(1-p)^{16} \times p$ for some p
A1 for awrt 0.0436

(c) 1st M1 Correct start to problem - sight or use of $B(n, 0.2)$. May be implied by $E(S) = 0.2n$
2nd M1 use of AP formula with $a = d = 2$, or 2xAP formula with $a = d = 1$, or equivalent
NB: must be working in another variable, AP formulae cannot be in terms of n
1st A1 $S^2 + S - n$ or equivalent, must be in a form from which expectation can be found
3rd M1 for use of $E(S) = 0.2n$ and $Var(S^2) = 0.16n$ (must be labelled or used as variance)
2nd A1 for correct value for $E(S^2)$
(*) 3rd A1 for a correct solution only, pulling together everything to get given answer

(d) 1st M1 for using a suitable prob statement or using a listing method to indicate first non-loss S
2nd M1 for solving the inequality or using a listing method to reach $P(S \geqslant 4)$
NB: award first two M marks for $P(S \geqslant 4)$ provided it does not come from incorrect working
3rd M1 for attempting $P(S \geqslant 4)$ with B(15, 0.2)
A1 for awrt 0.352
NB: solutions stemming from finding values of n gain no marks

(e) 1st M1 for sight or use of negB(r, 0.2) or sight of $r(r + 1)$
1st A1 for $r(r + 1) - T$, where T is defined
2nd M1 for use of $E(T) = 5r$ in an expression of "revenue" $- 5r$
2nd A1 for $r^2 - 4r$

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